# STRANGE OR EVEN STRANGER STARS? 

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#### Abstract

We describe the physical principles underlying the hypotheses on the existence of compact stars composed of strange quark matter or even more exotic matter. We review then the main parameters of hypothetical compact stars such as strange stars or even stranger stars (particularly, Q-stars). We formulate observational tests which would enable one to prove or disprove the existence of these objects and compare theoretical predictions with available observations.


Key words: compact stars; strange matter; Q-matter.

## 1. INTRODUCTION

Neutron stars, discovered by A. Hewish and S.J. Bell in 1967 as radio pulsars, have become common but most interesting astrophysical objects. Nowadays they are observed in all bands of electromagnetic spectrum, from radio to hard gamma-rays, in our Galaxy and outside. They manifest themselves in different ways (radio and X-ray pulsars, X-ray bursters, X-ray transients, etc.). They can be powerful emitters of neutrinos, gravitational waves, accelerated particles and pulsar winds.

There is no doubt that neutron stars are very compact. Their typical masses and radii are $M \sim 1.4 M_{\odot}$ and $R \sim 10-13 \mathrm{~km}$, respectively. Accordingly, neutron stars contain matter of supranuclear density $\rho \sim(2-15) \rho_{0}$ in their cores, where $\rho_{0} \approx 2.8 \times 10^{14} \mathrm{~g} \mathrm{~cm}^{-3}$ is the density of saturated nuclear matter. The properties of supranuclear matter cannot be tested in laboratory or calculated from first principles. These properties, first of all the composition and equation of state (EOS), remain currently uncertain and constitute the fundamental problem of compact stars (related to principal unsolved problems of nuclear physics, physics of strong interactions and particle physics). In this respect, compact stars can be considered as unique astrophysical laboratories to study the superdense matter.

Moreover, current theoretical and observational uncertainties are so large that one can question even the basic idea that compact stars observed (or at least some of
them) are indeed neutron stars. Contemporary theories cannot exclude the possibility that (some?) compact stars have other nature and can be, in fact, strange quark stars or even stranger stars. We will outline the main properties of such hypothetical stars. Our analysis will mostly be based on a recent monograph [1], where more details can be found. Let us mention also a detailed monograph [2] and several recent reviews [3, 4, 5].

## 2. NEUTRON STARS

### 2.1. Theory of internal structure

We will start with the summary of the neutron star physics (e.g., Ref. [1]). Current theories predict that a neutron star contains the atmosphere and four main internal regions: the outer crust, the inner crust, the outer core, and the inner core (Fig. 1).

The atmosphere is thin but important because thermal neutron star radiation is formed there. Cold or ultramagnetized neutron stars may have solid or liquid surface (without any atmosphere).

The outer crust is some hundred meters thick and extends from the atmosphere bottom to the layer of the density $\rho=\rho_{\mathrm{ND}} \approx 4 \times 10^{11} \mathrm{~g} \mathrm{~cm}^{-3}$. Its mass is $\sim 10^{-5} M_{\odot}$; its matter consists of ions $Z$ and electrons $e$. The electrons mainly constitute a strongly degenerate, ultrarelativistic, almost ideal gas, which gives the major contribution to pressure. The electron Fermi energy grows with increasing $\rho$, inducing beta captures in atomic nuclei and enriching the nuclei with neutrons. At $\rho=\rho_{\mathrm{ND}}$ the neutrons start to drip from the nuclei producing a free neutron gas.

The inner crust can be about one kilometer thick; its density varies from $\rho_{\mathrm{ND}}$ at the upper boundary to $\sim 0.5 \rho_{0}$ at the base; its mass is $\sim 10^{-2} M_{\odot}$. Its matter consists of electrons, free neutrons $n$, and neutron-rich atomic nuclei. The neutronization at $\rho \approx \rho_{\mathrm{ND}}$ greatly softens the EOS, but at the crust bottom the repulsive short-range component of the neutron-neutron interaction comes into play and introduces a considerable stiffness.

The outer core occupies the density range $0.5 \rho_{0} \lesssim \rho \lesssim 2 \rho_{0}$ and is several kilometers thick.


Figure 1. Schematic cross sections of a neutron star, a bare strange star and a strange star with crust.

Its matter consists of neutrons with several per cent admixture of protons $p$, electrons, and possibly muons $\mu$ (the so called $n р е \mu$ composition). All npe $\mu$-plasma components are strongly degenerate. The nucleons $N$ (neutrons and protons) form a strongly interacting Fermi liquid.

The inner core, where $\rho \gtrsim 2 \rho_{0}$, occupies the central regions of massive neutron stars (it does not occur in lowmass stars whose outer core extends to the very center). Its radius can reach several kilometers, and its central density can be as high as $(10-15) \rho_{0}$. Its composition and the EOS are very model dependent. Several hypotheses have been put forward, predicting the appearance of new fermions and/or boson condensates. The main four hypotheses are:
(1) The appearance of hyperons $(H)$, first of all $\Sigma^{-}$and $\Lambda$.
(2) Pion ( $\pi$ ) condensation - the appearance of a boson condensate of pion-like excitations with a strong renormalization and mixing of nucleon states.
(3) Kaon ( $K$ ) condensation - a Bose-Einstein condensation of kaon-like excitations which, like real kaons, possess strangeness.
(4) A phase transition to quark ( $q$ ) matter composed of deconfined light $u$ and $d$ quarks and strange $s$ quarks, and a small admixture of electrons, or even no electrons at all.

A new phase can appear via a first-order or a secondorder phase transition and is accompanied by the softening of the EOS. One also suggests the existence of mixed phases. Neutron stars with the cores containing essentially new phases of matter $(H, \pi, K, q)$ are sometimes called hybrid stars. Baryon component of matter in neutron star interiors ( $n, p$, hyperons, quarks) can be in superfluid state [6].

The EOS in a neutron star core cannot be reliably calculated because of the lack of the precise many-body theory of strongly interacting particles at the hadronic level. Instead, there are many theoretical models whose reliability
decreases with growing $\rho$ above $\rho_{0}$. These model EOSs can be divided into the soft, moderate and stiff with regard to the compressibility of dense matter. Table 1 lists eight EOSs (1-8) of nucleon-hyperon matter discussed in more details in Ref. [1]. For every EOS one can calculate a sequence of neutron star models parameterized by the central density $\rho_{c}$, with the circumferential stellar radius $R=R\left(\rho_{c}\right)$ and gravitational mass $M=M\left(\rho_{c}\right)$; this gives a line in the $M-R$ plane. A radial density distribution $\rho(r)$ within a neutron star depends on $M$. For example, the left panel of Fig. 2 shows density profiles for three neutron star models with $M=1.8,1.4$, and 0.5 $M_{\odot}$ calculated using the BBB2 EOS (No. 4) from Table 1. Fig. 3 presents $M-R$ diagrams for all eight EOSs $1-8$. The hatched domain is prohibited by General Relativity and by the requirement for the sound velocity in dense matter to be $v_{\text {sound }}<c$ (the domain is bound by $R=1.412 r_{\mathrm{g}}, r_{\mathrm{g}}=2 G M / c^{2}$ being the Schwarzschild radius; e.g., Ref. [1]). Neutron stars with $M \gtrsim M_{\odot}$ have radii $R \sim 9-13 \mathrm{~km}$. With increasing $\rho_{c}$ the star becomes more compact; there is the maximum mass limit $M_{\text {max }}$ for stable neutron stars (filled dots in Fig. 3). It varies from $M_{\max } \sim 1.4 M_{\odot}$ for the softest EOSs to $M_{\max } \sim 2.5 M_{\odot}$ for the stiffest ones. The EOSs can also be subdivided with respect to the composition of the matter (see above). Very stiff EOSs can possibly be attributed only to nucleon matter.

With decreasing $M$ below $\sim M_{\odot}$, neutron stars become less dense, and their radius $R$ grows up. There is the minimum mass limit $M_{\min } \sim 0.1 M_{\odot}$ of stable neutron stars (with $R\left(M_{\min }\right) \sim 300 \mathrm{~km}$ ). The theory of stellar evolution states that the birth of neutron stars with $M$ much below $M_{\odot}$ is highly unlikely.

### 2.2. Observational constraints

The EOS of dense matter can be constrained by comparing neutron star theory with observations. This can be done in many ways, particularly, using the $M-R$ diagram (Fig. 3). The most obvious way would be to accurately measure $M$ and $R$ for one or several neutron stars, which has not been done so far. Here we summa-

Table 1. Examples of EOSs employed

| No. | Composition | Ref. | EOS |
| :---: | :---: | :---: | :--- |
|  |  | Neutron | stars |
| 1 | $N$ | $[7]$ | BPAL [1] |
| 2 | $N H$ | $[8]$ | BGN1H1 [1] |
| 3 | $N$ | $[9]$ | FPS [1] |
| 4 | $N$ | $[10]$ | BBB2 [1] |
| 5 | $N$ | $[11]$ | SLy [1] |
| 6 | $N$ | $[8]$ | BGN1 [1] |
| 7 | $N$ | $[12]$ | APR [1] |
| 8 | $N$ | $[8]$ | BGN2 [1] |
|  |  | Strange | stars |
| A | SQM | $[13]$ | SQM1 [13] |
| B | SQM | $[13]$ | SQM2 [13] |
| C | SQM | $[14]$ | eos1 [14] |
|  |  | Q-stars |  |
| Q | Q-matter | $[15]$ | Q0, Q1, Q4, Q16 |

rize the most reliable and stringent current observational constraints (see Refs. [1, 5] for more details).

Masses of several radio pulsars in compact neutron star binaries have been measured with very high precision via pulsar timing. The most massive of them is the HulseTaylor pulsar B1913+16 with $M=1.4408 \pm 0.0006$ $M_{\odot}$ (at the $2 \sigma$ level) [16]. Its mass is compatible with any EOS $1-8$ in Fig. 3. There are several other binaries containing probably more massive neutron stars whose masses are less certain. The most promising is the radio pulsar J0751+1807 in a binary with a white dwarf. The pulsar mass is high but uncertain, $M(2 \sigma)=2.1_{-0.5}^{+0.4}$ $M_{\odot}$ [17]; the uncertainty will be greatly reduced in a few years. If its mass is really above $2 M_{\odot}$, it would strongly favor stiff EOSs (Fig. 3).

There have been numerous attempts to measure neutron star radii but the uncertainties of such measurements are still too large (e.g., Refs. [1, 5]).

It is important to mention the measurement of gravitational redshift $z=0.35$ of spectral lines of highly ionized iron and oxygen in radiation of the neutron star in the Xray binary EXO 0748-676 [18]. If the lines are formed at (near) the neutron star surface, the measured $z$ gives us a line in the $M-R$ diagram. Unfortunately, this line is consistent with any EOS 1-8 in Fig. 3.

A serious constraint could be made by observing rapidly spinning neutron stars. The fastest observed rotator PSR J1748-2446ad has the spin period $P=1.396 \mathrm{~ms}$ [19]. Fig. 3 shows the mass-shedding line for this pulsar. Its mass and radius should lie above this line in Fig. 3, which is again consistent with any EOS 1-8. An observation of faster rotation could be crucial. For instance, a discovery


Figure 2. Density profiles within ordinary neutron stars (left) and bare strange stars (right) of masses $M=0.5$, 1.4, and $1.8 M_{\odot}$. The $0.5 M_{\odot}$ neutron star has an extended crust (invisible in the figure) and $R=11.6 \mathrm{~km}$.
of a pulsar with $P=0.5 \mathrm{~ms}$ would rule out all EOSs $1-8$. Such a pulsar was discovered in 1989 in the supernova remnant 1987A, but the discovery turned out to be false [1].

Thus, current observations do not impose strong constraints on the EOS of dense matter, but such constraints can be obtained in the near future.

## 3. STRANGE STARS

As discussed in Sect. 2, neutron stars can have quark cores. We will not focus on this possibility but, instead, consider much more intriguing hypothetical strange stars which are entirely or almost entirely built of quark matter (called strange quark matter, SQM).

### 3.1. Strange quark matter

SQM consists of up ( $u$ ), down $(d)$, and strange ( $s$ ) quarks (no antiquarks) with a possible admixture of electrons (whose number density is $\sim 10^{-3}-10^{-4}$ of the quark number density). Let us remind that the quarks possess fractional electric charge ( $e_{q}=+\frac{2}{3} e,-\frac{1}{3} e,-\frac{1}{3} e$, for $q=u$, $d$, and $s$ ), and the $s$ quarks possess strangeness ( -1 ). All constituents of SQM are strongly degenerate fermions which obey the conditions of electric neutrality and beta equilibrium. Typical densities expected in strange stars are a few times of $\rho_{0}$, and typical quark Fermi energies are $\sim 500 \mathrm{MeV}$. Evidently, light $u$ and $d$ quarks (whose "current" masses are $m_{u} \sim m_{d} \sim 5-7 \mathrm{MeV}$ ) can be treated as massless, whereas the strange quark mass $m_{s} \sim 100-150 \mathrm{MeV}$ is non-negligible. Other quarks $(c, b, t)$ are too massive to be born in compact stars [20].

In contrast to hadronic matter, whose first-principle theory is absent at the hadronic level, there is the well formulated first-principle theory of quark matter - the quan-


Figure 3. Mass-radius diagram for eight sequences of nucleon-hyperon neutron stars (curves 1-8), three sequences of bare strange stars (curves $A, B$, and $C$ ) and one sequence of strange stars with crust (dashed curve Acrust, maximum crust density $\rho=\rho_{\mathrm{ND}}$ ). Maximum-mass configurations for every sequence are marked by filled circles. Dot-and-dashed curves are the best observational constraints. HT - accurately measured mass of the Hulse-Taylor pulsar PSR B1913+16; J0751 - measured central value of the mass for PSR J0751 +1807 (hatched region gives $2 \sigma$ errorbars); $z=0.35$ - gravitational redshift from the neutron star in EXO 0748-676; $P=1.4 \mathrm{~ms}$ - mass-shedding instability curve for the fastest observed pulsar J1748-2446ad; 0.5 ms - similar curve for a hypothetical neutron star with spin period $P=0.5 \mathrm{~ms}$ (see the text).
tum chromodynamics (QCD). This theory is practical and relatively simple in the asymptotic freedom regime, at quark energies much higher that 1 GeV , where the perturbative one-gluon-exchange approximation is applicable. QCD is also successful in explaining some properties of hadrons (composed of two or three low-energy confined quarks). Unfortunately, the first-principle QCD cannot currently describe a uniform quark matter (a many-quark system) at required energies $\sim 500 \mathrm{MeV}$; it cannot even prove the existence of such a matter.

In the absence of the exact solution, there are many models of quark matter. Some of them are based on the MIT bag model (e.g., Refs. [21, 22, 23, 24, 13]), but other models have also been employed, such as the model of density dependent quark masses and color-dependent interquark potential [14], and the Nambu-Jona-Lasinio model [25]. In all cases the respective EOSs are well approximated by a linear function,

$$
\begin{equation*}
P=a c^{2}\left(\rho-\rho_{\mathrm{s}}\right), \tag{1}
\end{equation*}
$$

where $a$ and $\rho_{\mathrm{s}}$ are model dependent constants. Thus, the pressure $P$ vanishes at $\rho=\rho_{\mathrm{s}}$ (that may be treated as the surface density of a strange star).

A linear EOS (1) is exact in the simplest MIT bag model of massless noninteracting $u, d, s$ quarks. In this model, the number densities, Fermi energies, and Fermi momenta are the same for all quark flavors, and the electrons are absent. In this case $a=1 / 3$, and $\rho_{\mathrm{s}}=4 B / c^{2}=$ $4.28 \times 10^{14} B_{60} \mathrm{~g} \mathrm{~cm}^{-3}, B$ being the bag constant, with $B_{60}=B /\left(60 \mathrm{MeV} \mathrm{cm}^{-3}\right)$. The bag constant is the main phenomenological parameter of the model which deter-
mines the excess energy of the QCD vacuum over the ordinary vacuum. The effects of quark interactions and finite $s$-quark mass violate the $\mathrm{SU}(3)$ symmetry and lead to the appearance of electrons. The form (1) becomes then inexact (though remains sufficiently accurate); $\rho_{\mathrm{s}}$ stays $\sim$ a few times of $\rho_{0}$.

The idea of the SQM hypothesis is simple. Let us take $P=0$ and calculate $e(u d s)=\rho_{\mathrm{s}} c^{2} / n_{\mathrm{b}}$, which is the energy density divided by the number density $n_{\mathrm{b}}$ of baryons which could be composed of quarks. For instance, we get $e(u s d)=837.3\left(B_{60}\right)^{1 / 4} \mathrm{MeV}$ for the bag model of massless noninteracting quarks. Let us compare $e(u d s)$ to the energy per baryon of an ordinary iron crystal, $e(\mathrm{Fe}) \approx 930.4 \mathrm{MeV}$. If $e(u d s)<e(\mathrm{Fe})$, then the quark matter is more stable than the ordinary matter even at $P=0$ and is energetically preferable over the ordinary matter. This condition may be fulfilled for a QCD model with a sufficiently weakly polarized vacuum (sufficiently low $B$ for a bag model). On the other hand, the QCD vacuum polarization cannot be too weak. Otherwise a given QCD model will be unable to explain the existence of hadrons (they would decay into free quarks in our ordinary world). If both requirements are fulfilled, then the quark matter represents the ground state of matter even at $P=0$. Then one can consider strange stars built entirely of quark matter, from the center to the surface, with the huge surface density, a few times of $\rho_{0}$. In the model of massless noninteracting quarks, these conditions reduce to $59 \mathrm{MeV} \mathrm{cm}^{-3} \leq B \leq 92 \mathrm{MeV} \mathrm{cm}^{-3}$, which is a reasonable range of $B$. In many other QCD models, MIT bag and non-bag ones, there are also ranges of parameters which allow for the existence of a selfbound quark


Figure 4. Adiabatic index versus density $\rho$ for the MIT bag model of massless non-interacting quarks with $B=$ $60 \mathrm{MeV} \mathrm{fm}{ }^{-3}$; $\rho_{c, \text { max }}$ is the central density of the maximum-mass strange star; $\gamma=4 / 3$ refers to a gas of ultrarelativistic free particles. From Ref. [1].
matter.
Notice, that the $u d s$ quark matter appears to be energetically preferable over the non-strange $u d$ quark matter. The existence of SQM does not preclude the existence of our ordinary hadronic world. The SQM can be more stable but it is separated from the ordinary matter by a huge potential barrier. The time of quantum tunneling through this barrier is much longer than the Universe age. There is no danger that our world will convert into the strange world (e.g., Ref. [1]).

The hypothesis of self-bound SQM came from the idea of Bodmer [26] published in 1971 on the existence of hypothetical "collapsed atomic nuclei," which could consist, in particular, of $u, d$, and $s$ quarks. Quantitative studies of "quark nuclei" were performed later [27], after the formulation of the MIT bag model [28, 29]. The idea on the existence of droplets of $u d s$ matter was proposed by Terazawa [30] but his paper was unnoticed at that time. SQM investigations really started after the publication of the seminal paper [21] by Witten in 1984. He clearly formulated the idea of self-bound quark matter in application to cosmological scenario (droplets) and strange stars. First model EOSs of SQM were constructed in [21, 22, 23, 24]. Current status of the EOS problem is reviewed in [1]-[5].

At $\rho \gg \rho_{\mathrm{s}}$ EOSs of SQM are not strongly different from EOSs of ordinary neutron star matter. The striking difference occurs at $\rho \rightarrow \rho_{\mathrm{s}}$, where the adiabatic index $\gamma=\mathrm{d} \ln P / \mathrm{d} \ln n_{\mathrm{b}}$ of SQM becomes very large (Fig. 4) indicating that SQM gets almost incompressible due to strong QCD binding.

### 3.2. Models of strange stars

One distinguishes bare strange stars and strange stars with crust (Fig. 1). The crust composed of normal electron-ion plasma can be accreted. Its maximum density cannot exceed $\rho_{\mathrm{ND}}$ but can be lower (Sect. 3.4).

Fig. 2 compares density profiles $\rho(r)$ in ordinary neutron
stars and bare strange stars of several masses. Neutron star models (the left panel) make use of the BBB2 EOS, with $M_{\max }=1.92 M_{\odot}$. Strange star models (the right panel) employ the SQM1 EOS based on the MIT bag (Table 1). It corresponds to $a=0.301$ and $\rho_{\mathrm{s}}=4.5 \times 10^{14}$ $\mathrm{g} \mathrm{cm}^{-3}$ in Eq. (1); $M_{\max }=1.80 M_{\odot}$.

Notice a striking difference between neutron stars and strange stars. Neutron stars have strongly heterogeneous structure with a huge ( $\sim 14-15$ orders of magnitude) density drop from the center to the surface. Their surface densities are ordinary for terrestrial conditions. Strange stars have much smaller density drops and enormous surface densities. In the maximum-mass strange star $\rho_{\mathrm{s}}$ is only $\sim 5$ times lower than $\rho_{c}$. In an $0.5 M_{\odot}$ strange star the density is almost constant; such a star is bound not by gravitation but by QCD forces.

Fig. 3 compares mass-radius tracks of bare strange stars (three EOSs) and neutron stars (eight EOSs). The main results of this comparison are as follows:
(I) Bare strange stars, in contrast to neutron stars, can have any small radius and mass (no minimum mass limit). Strange stars with $M \lesssim 0.5 M_{\odot}$ are nongravitating bodies built of incompressible SQM. Their mass $M \approx 4 \pi \rho_{\mathrm{s}} R^{3} / 3$ grows with increasing $R$ (while neutron star mass, typically, grows with decreasing $R$ ).
(II) Strange stars with $M \sim M_{\odot}$ can have small radii $R \lesssim 7-8 \mathrm{~km}$. This is the most important feature which distinguishes strange stars from neutron stars.
(III) At $M \gtrsim M_{\odot}$ the SQM becomes compressible and the $M-R$ dependence of strange stars mimics that for neutron stars. There is the maximum mass limit for strange stars; the masses $M_{\max }$ and radii $R\left(M_{\max }\right)$ of such stars are very model dependent (Fig. 3).

Because the EOS of SQM is nearly linear (1), there are many scaling relations which allow one to rescale parameters of strange stars within certain classes of EOSs (see, e.g., [1] and references therein). For instance, for MIT bag models of massless noninteracting quarks, which are characterized by a single parameter $B$,

$$
\begin{equation*}
M_{\max }=\frac{1.96}{\sqrt{B_{60}}} M_{\odot}, \quad R\left(M_{\max }\right)=\frac{10.71}{\sqrt{B_{60}}} \mathrm{~km} \tag{2}
\end{equation*}
$$

with $\rho_{\mathrm{c}, \text { max }}=2.06 \times 10^{15} B_{60}^{-1} \mathrm{~g} \mathrm{~cm}^{-3}$.
The mass-radius diagram for strange stars with crust is somewhat different. It is shown in Fig. 3 employing the SQM1 EOS and assuming the maximum crust (extending to $\rho=\rho_{\mathrm{ND}}$ ). Low-mass strange stars with crust are drastically different from low-mass bare strange stars; they have a small (non-gravitating) quark core and an extended crust bound to the core by gravitational forces. Strange stars with crust have minimum mass limit, $M_{\text {min }} \lesssim 0.01 M_{\odot}$ (with $R \sim 300 \mathrm{~km}$ ). They have also the minimum radius [31] ( $R \sim 7 \mathrm{~km}$ for the crust extended to $\rho_{\mathrm{ND}}$, Fig. 3), which is smaller than the radius of ordinary neutron stars. The crust of strange stars with $M \gtrsim M_{\odot}$ is too thin and low-massive to significantly modify the $M-R$ relation.

The effects of rotation on strange stars are reviewed, for instance, in Ref. [1]. In particular, there are many selfsimilarity relations for spinning bare strange stars. Some models of bare strange stars can sustain somewhat faster rotation than neutron stars.

The first models of neutron stars with quark cores were constructed in 1965 by Ivanenko \& Kurdgelaidze [32]. First models of stars entirely composed of quark matter were built [33] in 1970 by Itoh who used unrealistically high quark masses $(10 \mathrm{GeV})$ and obtained quark stars with $M_{\max } \sim 10^{-3} M_{\odot}$. First models of quark stars with the surface density $\rho_{\mathrm{s}} \sim \rho_{0}$ were built by Brecher \& Caporaso [34]. Models of strange stars based on the MIT bag model of massless noninteracting quarks were constructed by Witten in the seminal paper [21]. More realistic versions of the MIT bag model (with account for $s$-quark mass and quark interactions) were first employed in Refs. [23, 24]. The number of currently constructed strange star models is large.

### 3.3. Color superconductivity

Quark matter can be in superfluid (superconducting) state due to the attractive component of quark-quark interaction. The idea of such superfluidity was proposed [35] by Bailin \& Love (1984) who estimated the superfluid energy gap $\Delta \sim 1 \mathrm{MeV}$, which is of the same order of magnitude as the gap expected in the energy spectrum of nucleons in cores of ordinary neutron stars [6]. However, it has been realized later [36] that direct pairing color interaction between quarks is much stronger, than pairing interaction between color-neutral nucleons. Quark gaps can be much larger, $\Delta \sim 100 \mathrm{MeV}$, leading to huge critical temperatures $T_{c} \sim 5 \times 10^{11}$ for color superconductivity onset in quark matter.

Ref. [36] triggered a flow of publications on color superconductivity in quark matter (see [3,5] for recent references). This superconductivity turns out to be of different types. It can enforce color-flavor-locked phase of quark matter (pairing of $u s, u d$, sd quarks, with a common Fermi surface), possibly with no electrons at all. It can be two-flavor color superconductivity (pairing of $u d$ quarks). Color superconductivity can produce gaps in the energy spectra of pairing quarks at the Fermi surface, but it can also be the so called gapless superconductivity with gaps vanishing at certain values of quark momenta.

Luckily, according to estimates, even the presence of a huge gap $\Delta \sim 100 \mathrm{MeV}$ does not affect strongly the EOS of SQM and global parameters of strange stars. It can change the pressure of SQM [37] by $\sim\left(\Delta / \mu_{q}\right)^{2} \lesssim 5 \%$, for a typical quark chemical potential $\mu_{q} \sim 500 \mathrm{MeV}$. Nevertheless, color superconductivity strongly modifies kinetic properties and neutrino emission of SQM. Many such properties are still almost unexplored.


Figure 5. Mass-radius diagram for four sequences of Qstars (curves Q0, Q1, Q4, Q16), one sequence of bare strange stars (dashed curve B) and neutron stars (dash-and-dot curve 8). The shaded strip $J 0751$ is the same as in Fig. 3.

### 3.4. Surface structure and electric double layers

The surface of a bare strange star is most important because it regulates radiation of the star. The quark surface, determined by strong interactions, is very sharp; its thickness is $\sim 1 \mathrm{fm}$. The plasma frequency $\omega_{p q}$ of quark plasma is enormous; the respective energy is $\hbar \omega_{p q} \sim 20$ MeV , much higher than the expected thermal energy of SQM in strange stars. The intensity of thermal electromagnetic radiation within the SQM should be negligibly small, which greatly reduces the photon surface emission.

There are several drastically different models of surface layers of bare strange stars (see [5, 1] for more details).

The standard assumption is that the SQM contains an admixture of electrons. The electrons are bound to the SQM much weaker than quarks, by electromagnetic forces. The electrons can 'evaporate' from the quark matter and form a thin layer (called the electrosphere) above the quark surface. A charge separation within the electrosphere creates radial (outwardly directed) electric fields of double-layer type located within the electrosphere (negative electron layer overlaying SQM layer of a net positive charge). Calculations [24] give the electrosphere thickness $\sim 100 \mathrm{fm}$, and the electric field strength $\sim 3 \times 10^{17} \mathrm{~V} \mathrm{~cm}^{-1}$. Radiation from strange stars can be produced by some mechanisms involving quarks, electrons, and huge electric fields at the very quark surface and in the electrosphere. Radial electrospheric electric fields do not allow ions (atomic nuclei) falling onto the surface from the outside to penetrate under the surface and convert into the SQM.

A strange star with crust contains a layer of normal mat-
ter above the SQM. Such a star has a normal atmosphere and radiates as an ordinary neutron star. A thin layer of electrons is still present, but it fills the gap between the normal matter and the SQM. A huge repulsive Coulomb barrier (electric field $\sim 10^{17} \mathrm{~V} \mathrm{~cm}^{-1}$ ) shields the normal matter against conversion into the SQM. The density of the normal matter cannot exceed the neutron drip density. Otherwise dripped neutrons would trigger copious conversion of the normal matter into the SQM.

If the bulk SQM contains no electrons, the surface structure of a strange star can resemble the structure in the presence of electrons; the resemblance can be produced by the mass difference of $s$ and $u-d$ quarks (e.g., Ref. [1]). The surface layer can also consist of a crystal of SQM nuggets immersed in an electron gas [38].

### 3.5. Problems

The physics of strange stars is full of open problems. We just mention them; see review literature [1]-[5] for a more detailed discussion.

If strange stars exist, the basic problem is how they form and how they relate to neutron stars. The common wisdom is that the EOS in all compact stars should be obtained "from the same Hamiltonian," but it is consistent with many divergent ideas. Some authors state that all compact stars are neutron stars, while others claim that they are strange stars. In contrast, neutron and strange stars may represent two populations of compact stars. For instance, strange stars can be generally more massive and form in gravitational collapse of presupernovae with massive cores, or via conversion of neutron stars into strange stars (e.g., under the effect of accretion).

If all compact stars are strange stars, one should face the problem of pulsar glitches. These glitches are traditionally associated with neutron superfluid in inner crust of neutron stars, while strange stars have no inner crust. There is a growing evidence $[4,39]$ that some isolated neutron stars and neutron stars in soft X-ray transients in quiescent states emit thermal radiation from their surfaces. It would be a problem to explain this thermal emission from surfaces of bare strange stars. Thermal evolution of strange and neutron stars can be drastically different [4].

### 3.6. Observational evidence

There have been several claims of the discovery of strange stars, but in all the cases the observational evidence has appeared to be not decisive. The most distinct feature would be a small radius of a strange star. For instance, it was deduced [40] from X-ray observations of the nearby neutron star RX J1856-3754 that this star has the circumferential radius $R \lesssim 6 \mathrm{~km}$ and is actually a strange star. However, taking into account optical and UV observations gave larger, "neutron-star" values of $R$
(e.g., [41, 42]). When a rapidly spinning ( $P=0.5 \mathrm{~ms}$ ) pulsar was discovered in the supernova remnant 1987A, it was suggested to be a strange star [43] but the discovery turned out to be false.

In addition, many authors state that some observations are easier explained assuming that observed sources are strange stars rather than neutron stars. For example, it can be advantageous to suggest that superbursts of X-ray bursters occur in strange stars with crust [44].

There have been attempts to search for mini strange stars - droplets of SQM - in space [45] but without definite result. Thus, there are tentative indications for the existence of SQM and strange stars but no direct proof.

## 4. STRANGER STARS?

Needless to say, theoretical predictions go beyond SQM and strange stars. Several others types of exotic (even stranger, particularly self-bound) dense matter have been suggested which can be constituents of even stranger stars (reviewed, e.g., in Ref. [1]). These predictions have much more speculative theoretical and experimental basis.

For example, we mention hypothetical self-bound Qmatter and respective Q -stars [15]. The Q -matter is predicted by sophisticated supersymmetric extensions of the Standard Model of elementary particles and their interactions; the matter contains a scalar field condensate with well defined nucleonic quantum numbers. An EOS of Qmatter can be determined by the energy density $U_{0}$ of the condensate field and a dimensionless parameter $\zeta$ which is a combination of $U_{0}$ and a coupling constant of nucleons within the matter. Bahcall et al. [15] considered four model EOSs, with $\zeta=0,1,4$, and 16 (and $U_{0}=13 \mathrm{MeV}$ $\mathrm{fm}^{-3}$ in all cases). We denote these EOSs as Q0, Q1, Q4, and Q16, respectively (Table 1). They are well approximated by Eq. (1), but the condensate field makes nucleons in the Q-matter almost massless, so that $\rho_{\mathrm{s}}$ is noticeably lower than in SQM. The $M-R$ diagrams for Q -stars with EOSs Q1-Q16 are presented in Fig. 5. For comparison, we plot also one curve for neutron stars (EOS BGN2) and one for strange stars (EOS SQM2), as well as observational mass limits for PSR J0751+1807. We see that Q-stars can have masses and radii much higher than neutron stars or strange stars (up to $M \sim 8 M_{\odot}$ and $R \sim 35 \mathrm{~km}$ for the Q16 EOS), but the justification of these models is questionable.

## 5. CONCLUSIONS

1. Strange or even stranger stars are not forbidden by the laws of Nature, and, hence, should be studied.
2. Are these stars currently needed to explain observations? - No and yes! No, because there is no
strict direct evidence for their existence. Yes, because there are some indirect indications, and because any realistic alternative to neutron stars has to be analyzed.
3. From theoretical point of view, strange stars are the best alternative to neutron stars, being based on the well defined theoretical ground (QCD).
4. Today QCD is helpless at the densities expected in compact stars; it can neither prove nor disprove the existence of strange stars, but provides many divergent theoretical models. Further development of strict QCD methods would be most helpful.
5. To prove the existence of strange or even stranger stars from observations, one should look for compact stars which cannot be neutron stars (small or large radii, large masses, superfast rotation, etc.). New most exciting discoveries are ahead.

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