New results in the theory of present dark energy

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Amount of light neutrino-like components in the Universe

Cosmological upper limit on neutrino masses

$f(R)$ gravity

Massive neutrinos with $f(R)$ gravity

Conclusions
Amount of light neutrino-like components in the Universe

**Light** – (practically) massless at the time of BBN.

If conventionally expressed in terms of effective neutrino types:

\[
\rho_r = \rho_\gamma \left[ 1 + N_{\text{eff}} \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \right]
\]

with \( N = 3.046 \) for the standard three flavor neutrino species.

**BBN** (Izotov and Thuan, 2010): \( N_{\text{eff}} = 3.7^{+0.8}_{-0.7} \) (2\( \sigma \)).

**CMB+BAO+H_0** (Keisler et al., 2011): \( N_{\text{eff}} = 3.86 \pm 0.42 \).

**Clusters+CMB+BAO+H_0** (Burenin and Vikhlinin, arXiv:1202.2889): \( N_{\text{eff}} < 4.6 \) (95\% CL).

Conclusion: one additional (sterile) neutrino species is well possible.
Cosmological upper limit on neutrino masses

(Absence of) damping of the power spectrum of density perturbations at small scales.

For three standard neutrino species:
BAO(WiggleZ)+CMB (Riemer-Sorensen et al., arXiv:1112.4940): \( \sum_i m_{\nu i} < 0.29 \text{ eV} \).
Clusters+CMB+BAO+H_0+SN (Burenin and Vikhlinin, arXiv:1202.2889): \( \sum_i m_{\nu i} < 0.28 \text{ eV} \).

For larger number of species the upper limit becomes weaker but not too much. E.g. Burenin and Vikhlinin give \( \sum_i m_{\nu i} < 0.72 \text{ eV} \).

Conclusion: in the scope of the Einstein gravity, it is not possible to have a sterile neutrino with the standard number density and the restmass \( m \gtrsim 1 \text{ eV} \).
Hints from ground-based experiments

Hints for possibility of existence of light sterile neutrinos with \( m \sim 1 \text{eV} \).

1. Neutrino oscillation experiments such as LSND and MiniBooNE.
2. The Gallium anomaly of SAGE and GALLEX experiments.
3. The reactor antineutrino anomaly.

See the white paper K. N. Abazajian et al., arXiv:1204.5379 for details.
$f(R)$ gravity

The simplest model of modified gravity considered as a phenomenological macroscopic theory in the fully non-linear regime and non-perturbative regime. It can produce models of present geometrical dark energy alternative to a cosmological constant.

$$S = \frac{1}{16\pi G} \int f(R)\sqrt{-g} \, d^4x + S_m$$

$$f(R) = R + F(R), \quad R \equiv R^\mu_{\mu}.$$  

Here $f''(R)$ is not identically zero. Usual matter described by the action $S_m$ is minimally coupled to gravity.
Field equations

\[
\frac{1}{8\pi G} \left( R^\nu_\mu - \frac{1}{2} \delta^\nu_\mu R \right) = - \left( T^\nu_\mu_{vis} + T^\nu_\mu_{DM} + T^\nu_\mu_{DE} \right),
\]

where \( G = G_0 = const \) is the Newton gravitational constant measured in laboratory and the effective energy-momentum tensor of DE is

\[
8\pi G T^\nu_\mu_{(DE)} = F'(R) R^\nu_\mu - \frac{1}{2} F(R) \delta^\nu_\mu + \left( \nabla_\mu \nabla^\nu - \delta^\nu_\mu \nabla_\gamma \nabla^\gamma \right) F'(R).
\]

Because of the need to describe DE, de Sitter solutions in the absence of matter are of special interest. They are given by the roots \( R = R_{ds} \) of the algebraic equation

\[
Rf'(R) = 2f(R).
\]
Models of present dark energy in $f(R)$ gravity

Much more difficult to construct than inflationary models (the $R + R^2$ model and close ones).

An example of the viable model satisfying viability conditions in the present Universe (Starobinsky, 2007):

$$f(R) = R + \lambda R_0 \left( \frac{1}{\left(1 + \frac{R^2}{R_0^2}\right)^n} - 1 \right)$$

with $n \geq 2$. $f(0) = 0$ is put by hand to avoid the appearance of a cosmological constant in the flat space-time. Similar models: Hu and Sawicki, 2007; Appleby and Battye, 2007.

No good microscopic justification for both the energy scale and the complicated form of $f(R)$ needed ($0 < f' < 1$).
Anomalous growth of perturbations in $f(R)$ gravity

Occurs deeply in the sub-horizon regime when the scalaron becomes light (its Compton length exceeds the comoving scale of perturbations):

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}} \rho \delta = 0, \quad G_{\text{eff}} = \frac{G}{f'} \frac{1 + 4 \frac{k^2}{a^2} \frac{f''}{f'}}{1 + 3 \frac{k^2}{a^2} \frac{f''}{f'}}.$$
Massive neutrinos with $f(R)$ gravity

The anomalous growth of perturbations may be partially compensated by an increase of $\sum_{\nu} m_{\nu}$ as compared to the standard $\Lambda CDM$, up to $\mathcal{O}(0.5 \text{ eV})$ in the case of 3 standard neutrinos.

More interesting results for additional sterile neutrinos (assumed to be significantly more massive than the standard ones).

H. Motohashi, A. A. Starobinsky, J. Yokoyama, arXiv:1203.0805
Comparison of the $f(R)$ and the standard $\Lambda CDM$ models in case of one massive sterile neutrino with $m = 1 \text{ eV}$. 
In this case, the difference of best-fit $\chi^2$ is $\chi^2_{\Lambda CDM} - \chi^2_{fR} = 11.0$ and for $f(R)$ cosmology $\sigma_8 = 0.816^{+0.09}_{-0.06}$.

In the case of two sterile neutrinos, $\sigma_8$ is $\sim 10\%$ less than for one sterile neutrino that presents the possibility to distinguish these two scenarios, e.g. using cluster abundance, and favors the latter case.
Conclusions

- In contrast to the standard $\Lambda CDM$ model, cosmology based on $f(R)$ gravity admits one or two massive sterile neutrino species with masses $m_{\nu} \sim 1$ eV and removes the problem of inadmissibly low value of $\sigma_8$.

- The case of one massive sterile neutrino gives a better value for $\sigma_8$ than that with two massive ones.