Neutron star (NS) thermal evolution is driven by cooling via surface electromagnetic, neutrino, and neutrino emission from their interior, and by heating due to, in particular, decay of internal magnetic field $B$. This effect is well-studied in the NS crust (e.g. [1]) but is less explored in the core, where $B$ dissipation mainly depends on the underlying microscopic processes (for example, in the framework of neutrino processes), frictional heating due to deformation of various components of the fluid, and (if neutrons are paired) scattering of various particle species via the $s$-fluctuations. For the simplest model of the normal NS core, we [2] studied this heating in a quasi-stationary approximation to the core magnetohydrodynamics (MHD). Here we extend this approach to the full $s$-neutrons core composition. We also consider neutron pairing, applying methods of our work [3] to derive the magnetic dissipation rate. We use non-linear MHD but follow a linear approximation of state (EoS) model. We neglect NS rotation, and linearize MHD equations, assuming that electric currents producing $B$ are a small perturbation of the non-magnetically symmetric hydrostatic equilibrium. We employ Cowling approximation. Finally, we use the quasi-stationary approach, i.e. drop the $\partial / \partial t$ terms in all equations but the Faraday law. But we also assume the core isothermal with temperature $T$.

4. Superconducting $p$, normal $n$

Neutrons and protons in the NS core may be superfluid, their (local) critical temperature $T_{c}$ is the superconducting transition temperature $T_{c}$ and $\Delta H_{c2}$ and $\Delta H_{c1}$ are the critical magnetic fields. We consider protons to be a type II superconductor with critical fields $H_{c2}$ and $H_{c1}$. In this section, we assume $T_{c} < T < T_{c2}$ (thermal excitations of $p$ fluid are negligible) and $B = B_{c1}$. The system of quasi-stationary MHD equations has 3 additional terms for the core temperature $T$ and the averaged magnetic field in the core. The Clebsch-Coulomb interaction term for $p$ and $n$ contributes to the core energy.

\[ \frac{d\log T}{dt} = - \frac{\Delta H_{c1} - \Delta H_{c2}}{T_{c} - T} \]

The superconductivity $p$ breaks $B$ configuration, $B_{c2} \neq 0$.

5. Estimates for the case of superconducting $p$

We assume the single proton superconductivity with $T_{c} = 5 \times 10^{3}$ K throughout the NS core. In this typical critical fields are $H_{c2} \sim 10^{15}$ G, $H_{c1} \sim 10^{14}$ G, and for a long-range timescale $\Delta H_{c1}$ is a negligible. The total energy dissipation is $w_{p}$

\[ w_{p} = \frac{\Delta H_{c1} - \Delta H_{c2}}{T_{c} - T} (\frac{d\log T}{dt}) \]

Using the results of Sects. 3.5.6, we can estimate the $B$ dissipation rate (and the corresponding heating) in the NS core for T and some “average” EoS. $\xi_{5}$ is the upper estimate of the heating rate for $B_{c2} \neq 0$, and $\xi_{10}$ is the lower estimate for $B_{c2} = 0$.

\[ \frac{d\log B}{dt} = \frac{\Delta H_{c1} - \Delta H_{c2}}{T_{c} - T} \]