# Magnetic field dissipation in nucleonic neutron star cores

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## Abstract

**We study the long-term heating due to magnetic field decay in the core of neutron star. Three cases of nucleonic core are considered: (i) nucleons are normal; (ii) neutrons are normal but protons are strongly superconducting; (iii) both neutrons and protons are strongly paired. We explore efficiencies of various processes responsible for the magnetic field evolution. For the most important of them, we give simple scaling relations (depending on the internal stellar temperature and the averaged magnetic field in the core) to estimate the heating rate due to the magnetic field decay. Comparison to properties of observed neutron stars suggests that such heating is (at least partially) responsible for the thermal states of middle-aged magnetars and highly-magnetized isolated neutron stars with ages of 1 — 10 Myr.**

## 1. Introduction

Neutron star (NS) thermal evolution is driven by cooling via surface electromagnetic emission and neutrino emission from their interiors, and by heating due to, in particular, decay of internal magnetic field $\boldsymbol{B}$. This effect is well-studied in the NS crust (e.g. [1]) but is less explored in the core, where $\boldsymbol{B}$ dissipates mainly due to nonequilibrium Urca processes, mutual friction of various components of the fluid, and (if nucleons are paired) scattering of various particle species off $p$-fluxtubes. For the simplest model of the normal $npe$ core, we [2] studied this heating in a quasistationary approach to the core magnetohydrodynamics (MHD). Here we extend this approach to the full $npe\mu$ core composition. We also consider nucleon pairing, applying methods of our work [3] to derive the magnetic dissipation rate. We use nonrelativistic MHD but employ a realistic equation of state (EoS) model. We neglect NS rotation, and linearize MHD equations, assuming that electric currents producing $\boldsymbol{B}$ are a small perturbation of the nonmagnetic spherically symmetric hydrostatic equilibrium. We use Cowling approximation. Finally, we use the quasistationary approach, i.e. drop the $\partial/\partial t$ terms in all equations but the Faraday law. We also assume the core isothermal with temperature $T$.

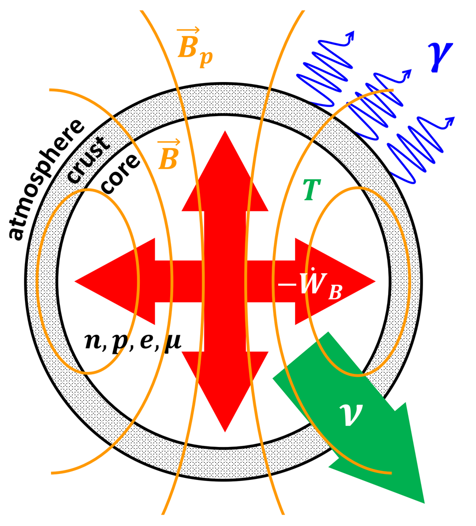


Figure 1: schematic NS structure.

## 2. Quasistationary $B$ evolution in normal NS

Quasistationary MHD equations for normal core matter are [4, 2] the Faraday law $\partial\boldsymbol{B}/\partial t = -c\,\mathrm{curl}\,\boldsymbol{E}$, accompanied by the equations

$$\mathrm{div}\,(n_a\boldsymbol{u}_a) = \Delta\Gamma_a, \quad \sum_a e_a n_a = 0, \quad \sum_a e_a n_a \boldsymbol{u}_a = \frac{c}{4\pi}\,\mathrm{curl}\,\boldsymbol{B}, \tag{1a}$$

$$\sum_{b\neq a} J_{ab}\,(\boldsymbol{u}_a - \boldsymbol{u}_b) = -n_a\nabla\delta\mu_a + e_a n_a\left(\boldsymbol{E} + \frac{\boldsymbol{u}_a}{c}\times\boldsymbol{B}\right). \tag{1b}$$

Here $e_a$, $n_a$, $\delta\mu_a$ and $\boldsymbol{u}_a$ are the electric charge, number density, chemical potential perturbation, and velocity for particles $a = n, p, e, \mu$ (notice that in the linear approximation $n_a$ is an unperturbed spherically symmetric density); $\Delta\Gamma_a$ is the production rate for species $a$ due to non-equilibrium reactions; $J_{ab} = J_{ba}$ are the friction coefficients; $\boldsymbol{E}$ is the electric field (with the equilibrium background subtracted).

The total magnetic energy dissipation in the core volume is

$$-\dot{W}_B = \int \mathrm{d}V\,\frac{\boldsymbol{B}}{4\pi}\frac{\partial\boldsymbol{B}}{\partial t} = \int \mathrm{d}V\left[\sum_a \delta\mu_a\Delta\Gamma_a + \frac{1}{2}\sum_{a,b} J_{ab}\,(\boldsymbol{u}_a - \boldsymbol{u}_b)^2\right]. \tag{2}$$

Here the surface terms are neglected (our estimates indicate they are small). For $npe\mu$ core composition, $\Delta\Gamma_p = -\Delta\Gamma_n = \Delta\Gamma_e + \Delta\Gamma_\mu$. Introducing $\Delta\mu_\ell = \delta\mu_p + \delta\mu_\ell - \delta\mu_n$, $\ell = e, \mu$, and assuming the linear regime, $\Delta\mu_\ell \ll \pi k_B T$, one has $\Delta\Gamma_\ell = -\lambda_\ell(T)\Delta\mu_\ell$. The reaction rates $\lambda_\ell(T) > 0$ are provided by non-equilibrium Urca reactions [5].

## 3. Estimate of dissipation rate in normal NS core

To illustrate the core microphysics, we use the BSk24 EoS [6] and take the friction coefficients from [7] (fig. 2). They scale with temperature as $J_{np}, J_{n\ell} \propto T^2$, $J_{p\ell}, J_{e\mu} \propto T^{5/3}$. The modified Urca rates $\lambda_{e,\mu} \propto T^6$ are taken from [5].

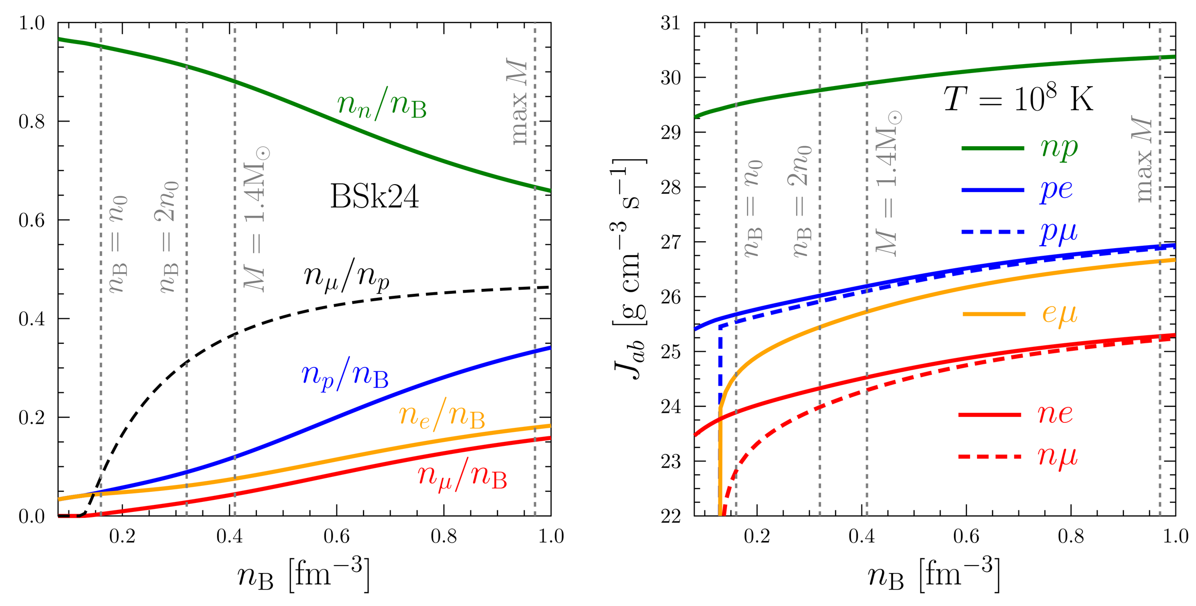


Figure 2: particle fractions *(left)* and friction coefficients in eq. (1b) *(right)* for BSk24 EoS. In the right plot, $T = 10^8$ K is assumed but $n$- and $p$-pairing is neglected.

Instead of a rigorous calculation of $\dot{W}_B$, we roughly estimate the terms in the integrand of eq. (2). We consider the $J_{ab}$ and $\lambda_\ell$ values at $n_B = 2n_0$ being typical. From eq. (1) we have

$$n_B\nabla\delta\mu_n + n_e\nabla\Delta\mu_e + n_\mu\nabla\Delta\mu_\mu = (4\pi)^{-1}\,\mathrm{curl}\,\boldsymbol{B}\times\boldsymbol{B}, \tag{3}$$

and thus there is a scaling $|\nabla\delta\mu_a| \propto B^2/L$, where $B$ is some typical value of the magetic field in the core, and $L$ is a typical lengthscale of $\boldsymbol{B}$ spatial variation. Further, using that $J_{n\ell} \ll J_{e\mu} \lesssim J_{p\ell} \ll J_{np}$, for typical magnetar conditions we have

$$\boldsymbol{u}_p - \boldsymbol{u}_n = n_n\nabla\delta\mu_n/J_{np}, \quad |\boldsymbol{u}_p - \boldsymbol{u}_\ell|,\ |\boldsymbol{u}_e - \boldsymbol{u}_\mu| \propto |\nabla(\Delta\mu_\mu - \Delta\mu_e)|/J_{p\mu}. \tag{4}$$

Together with eq. (2) this yields $\dot{W}_B \approx -H_R - H_{np} - H_{pe\mu}$, where

$$H_R = \int \mathrm{d}V\left(\lambda_e\Delta\mu_e^2 + \lambda_\mu\Delta\mu_\mu^2\right) \sim 10^{24}B_{14}^4T_8^6\,\mathrm{erg\,s^{-1}}, \tag{5a}$$

$$H_{np} = \int \mathrm{d}V\,(n_n\nabla\delta\mu_n)^2/J_{np} \sim 10^{30}B_{14}^4T_8^{-2}L_6^{-2}\,\mathrm{erg\,s^{-1}}, \tag{5b}$$

$$H_{pe\mu} = \int \mathrm{d}V\left[J_{pe}(\boldsymbol{u}_p - \boldsymbol{u}_e)^2 + J_{p\mu}(\boldsymbol{u}_p - \boldsymbol{u}_\mu)^2 + J_{e\mu}(\boldsymbol{u}_e - \boldsymbol{u}_\mu)^2\right] \tag{5c}$$

describe dissipation due to, respectively, Urca processes, $np$ friction, and friction of $p, e, \mu$ against each other. Here $B_{14} = B/(10^{14}\,\mathrm{G})$, $T_8 = T/(10^8\,\mathrm{K})$, $L_6 = L/(10^6\,\mathrm{cm})$. Estimate of the latter term is ambiguous, but we expect $H_{pe\mu} \ll (H_R, H_{np})$ except a narrow range $T \sim 10^9$ K, where $H_R \sim H_{np}$.

## 4. Superconducting $p$, normal $n$

Neutrons and protons in the NS core can be superfluid, their (local) critical temperatures are $T_{cn}$ and $T_{cp}$. We consider protons to be a type II superconductor with critical fields $H_{c1}$ and $H_{c2}$. In this section, we assume $T_{cn} < T \ll T_{cp}$ (thermal excitations of $p$ fluid are negligible) and $|\boldsymbol{B}| < H_{c2}$. Then the system of quasistationary MHD equations is [3, 8]

$$\mathrm{div}\,(n_a\boldsymbol{u}_a) = 0, \quad \sum_a e_a n_a = 0, \quad \sum_a e_a n_a\boldsymbol{u}_a = 0, \quad \boldsymbol{b} = \boldsymbol{B}/|\boldsymbol{B}| \tag{6a}$$

$$\sum_{b\neq a}\boldsymbol{\mathcal{F}}_{ab} - \frac{\delta_{ap}}{4\pi}\,\mathrm{curl}\,\boldsymbol{H}_{c1}\times\boldsymbol{B} = -n_a\nabla\delta\mu_a + e_a n_a\left(\boldsymbol{E} + \frac{\boldsymbol{u}_a}{c}\times\boldsymbol{B}\right), \tag{6b}$$

$$en_p\frac{\boldsymbol{u}_p - \boldsymbol{V}_L}{c}\times\boldsymbol{B} + \frac{1}{4\pi}\,\mathrm{curl}\,\boldsymbol{H}_{c1}\times\boldsymbol{B} + \sum_{a\neq p}\boldsymbol{\mathcal{F}}_{ap} = 0, \quad \boldsymbol{H}_{c1} = H_{c1}\boldsymbol{b}. \tag{6c}$$

Here $\delta_{ab}$ is the Kronecker delta, $\boldsymbol{V}_L \perp \boldsymbol{B}$ is the velocity field of the proton fluxtubes, $\boldsymbol{u}_p$ should be treated as a velocity of the proton superfluid, and $\boldsymbol{\mathcal{F}}_{ab} = -\boldsymbol{\mathcal{F}}_{ba}$ is the force the species "$a$" acts on the species "$b$". Namely,

$$\boldsymbol{\mathcal{F}}_{ab} = J_{ab}\,(\boldsymbol{u}_a - \boldsymbol{u}_b), \quad \boldsymbol{\mathcal{F}}_{ap} = D_a\,(\boldsymbol{u}_a - \boldsymbol{V}_L)_\perp, \quad \text{for } a, b = n, e, \mu. \tag{7}$$

$\boldsymbol{x}_\perp = \boldsymbol{x} - \boldsymbol{b}(\boldsymbol{b}\cdot\boldsymbol{x})$. The electric field (with the equilibrium background subtracted again) could be decomposed as $\boldsymbol{E} = -(\boldsymbol{V}_L/c)\times\boldsymbol{B} - \nabla\varphi$, and thus the Faraday law is $\partial\boldsymbol{B}/\partial t = \mathrm{curl}(\boldsymbol{V}_L\times\boldsymbol{B})$.

The total energy dissipation is (we neglect surface terms again)

$$-\dot{W}_B = \int \mathrm{d}V\frac{\boldsymbol{H}_{c1}}{4\pi}\frac{\partial\boldsymbol{B}}{\partial t} = \int \mathrm{d}V\left[\sum_{a\neq p}D_a(\boldsymbol{u}_a - \boldsymbol{V}_L)_\perp^2 + \frac{1}{2}\sum_{a,b\neq p}J_{ab}(\boldsymbol{u}_a - \boldsymbol{u}_b)^2\right]. \tag{8}$$

## 5. Estimates for the case of superconducting $p$

We assume the singlet proton superconductivity with $T_{cp} = 5\times 10^9$ K throughout the NS core. In this case typical critical fields are $H_{c1} \sim 10^{14} - 10^{15}$ G, $H_{c2} \sim 10^{15} - 10^{16}$ G [see fig. 3*(left)*]. The coefficients $J_{n\ell} \propto T^2$ are taken from [7], and $D_\ell \propto B$ from [9]. We use "normal" expression [7] for $J_{e\mu} \propto T^{5/3}$ as an upper estimate. To estimate $D_n$, we assume that it is provided by scattering $n$ off the unpaired cores of $p$ fluxtubes. Then

$$D_n \sim J_{np}\times\frac{\text{volume of fluxtube cores}}{\text{total volume}} \sim J_{np}\frac{B}{2H_{c2}} \propto BT^2. \tag{9}$$

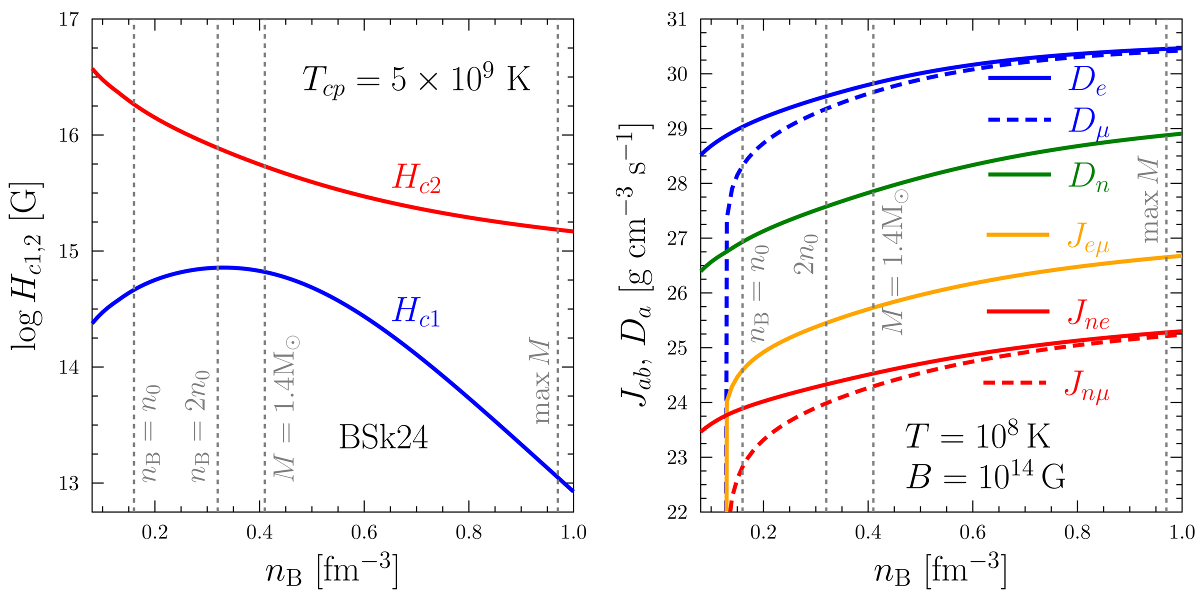


Figure 3: critical fields for *(left)* and dissipative coefficients *(right)* in the $npe\mu$ matter with $T_{cp} = 5\times 10^9$ K, $T \ll T_{cp}$, for BSk24 EoS.

Again, to estimate $\dot{W}_B$ we consider the values of $J_{ab}$ and $D_a$ at $n_B = 2n_0$ as typical ones. From eq. (6b) we have

$$n_B\nabla\delta\mu_n + n_e\nabla\Delta\mu_e + n_\mu\nabla\Delta\mu_\mu = (4\pi)^{-1}\,\mathrm{curl}\,\boldsymbol{H}_{c1}\times\boldsymbol{B}, \tag{10}$$

and thus there is a scaling $|\nabla\delta\mu_a| \propto H_{c1}B/L$, where $B$ and $H_{c1}$ are some typical values in the core, and $L$ is a lengthscale of $\boldsymbol{H}_{c1}$ variation. Further, for a typical NS conditions we have $J_{n\ell} \ll (J_{e\mu}, D_n) \ll D_\ell$ [see fig. 3*(right)*]. Since the coefficients $D_a$ describe the force that does not act along the field lines, the solution to eqs. (6) is essentially different in $\boldsymbol{B}\|$- and $\boldsymbol{B}\perp$-directions. To estimate the terms in eq. (8) we can write approximately

$$|\boldsymbol{u}_n - \boldsymbol{V}_L|_\perp \propto |\nabla\delta\mu_n|_\perp/D_n, \quad |\boldsymbol{u}_\ell - \boldsymbol{V}_L|_\perp \propto |\nabla\Delta\mu_\ell|_\perp/(en_\ell B/c), \tag{11a}$$

$$|\boldsymbol{u}_\ell - \boldsymbol{u}_n| \propto |\nabla\delta\mu_n|/J_{n\ell}, \quad |\boldsymbol{u}_e - \boldsymbol{u}_\mu| \propto |\nabla(\Delta\mu_\mu - \Delta\mu_e)|/J_{e\mu}. \tag{11b}$$

For an arbitrary $\boldsymbol{B}$ configuration, $\dot{W}_B$ appears to be governed by $e\mu$ friction along the field lines,

$$-\dot{W}_B \approx H_{e\mu} = \int \mathrm{d}V\,J_{e\mu}(\boldsymbol{u}_e - \boldsymbol{u}_\mu)^2 \sim 10^{36}B_{14}^2T_8^{-5/3}L_6^{-2}\,\mathrm{erg\,s^{-1}}. \tag{12a}$$

Such intensive dissipation (enhanced by more rapid non-dissipative $\boldsymbol{B}$ evolution) should [3] lead to rapid field rearrangement to a configuration that suppress the huge term (12a) in $\dot{W}_B$. This process goes simultaneously with the NS cooling down to $T \ll T_{cp}$. After the field is rearranged, the term corresponding to lepton friction off fluxtubes becomes dominating,

$$-\dot{W}_B \approx H_{\ell L} = \int \mathrm{d}V\sum_{\ell=e,\mu}D_\ell(\boldsymbol{u}_\ell - \boldsymbol{V}_L)_\perp^2 \sim 10^{29}B_{14}L_6^{-2}\,\mathrm{erg\,s^{-1}}. \tag{12b}$$

Notice that (as our estimates suggest) $n\ell$ and $n$-fluxtube friction make smaller contributions to $\dot{W}_B$.

## 6. Superconducting $p$, superfluid $n$

Typical $n$ critical temperature is $T_{cn} \sim$ *few* $\times 10^8\,\mathrm{K} < T_{cp}$. Thus for old and cold NSs it is instructive to consider the case $T \ll T_{cn}$, when both $n$ and $p$ are strongly paired. In this case quasistationary MHD equations are given in [3] (however, entrainment effects are neglected). They are similar to eqs. (6), (7), but now $\boldsymbol{u}_n$ is the velocity of neutron superfluid, and $J_{n\ell} = D_n = 0$. Corresponding expression for $\dot{W}_B$ can be obtained from eq. (8) with the same changes. As shown in [3], in this case $\delta\mu_n = 0$, but $\Delta\mu_\ell$, $\boldsymbol{u}_\ell - \boldsymbol{V}_L$ and $\boldsymbol{u}_e - \boldsymbol{u}_\mu$ obey the same estimates as for normal $n$ and paired $p$. Thus $\boldsymbol{B}$ dissipates similarly to the case of unpaired $n$, and for a long-range timescale the estimate (12b) is still valid for $\dot{W}_B$.

## 7. Comparison to observations

Using the results of Secs. 3,5,6, we can estimate the $\boldsymbol{B}$ dissipation rate (and thus the magnetic heating) in the NS core with given $T$ and some "averaged" $B$. Eqs. (5a,5b) and (12b) provide expressions for $\dot{W}_B$ in the limits of either normal or strongly paired $p$. In an intermediate case $\dot{W}_B$ should be between these limits. Below we consider two options to (artificially) interpolate between the normal and superfluid $\dot{W}_B$: *(a)* an upper estimate of the heating rate

$$-\dot{W}_B = \max\left\{H_R + H_{np}, H_{\ell L}\right\}; \tag{13a}$$

*(b)* a smooth glueing of normal and superfluid estimates

$$-\dot{W}_B = g\left(B/H_{c2}(T)\right)(H_R + H_{np}) + \left[1 - g\left(B/H_{c2}(T)\right)\right]H_{\ell L}. \tag{13b}$$

The monotonic function $g(x)$ is equal to 1 at $x \geqslant 1$ and smoothly tends to 0 at $x \ll 1$ (actually, at $B < H_{c1}(T)$). It is instructive to consider the "heating ratio" $h = (-\dot{W}_B)/(L_\nu + L_\gamma)$. The neutrino luminosity $L_\nu$ is considered to be due to $nn$-bresstrahlung [5] (even partially paired $p$ suppress standard modif. Urca $\nu$ emission). The photon luminosity $L_\gamma$ is related to internal $T$ by some heat blanketing envelope model [10]. In fig. 4 we plot $h$ based on eq. (13b) via the pastel color map (solid lines for $h = const$ levels). Dashed lines show $h = const$ levels for $h$ based on eq. (13a). In both cases, we take $L \sim 3$ km.

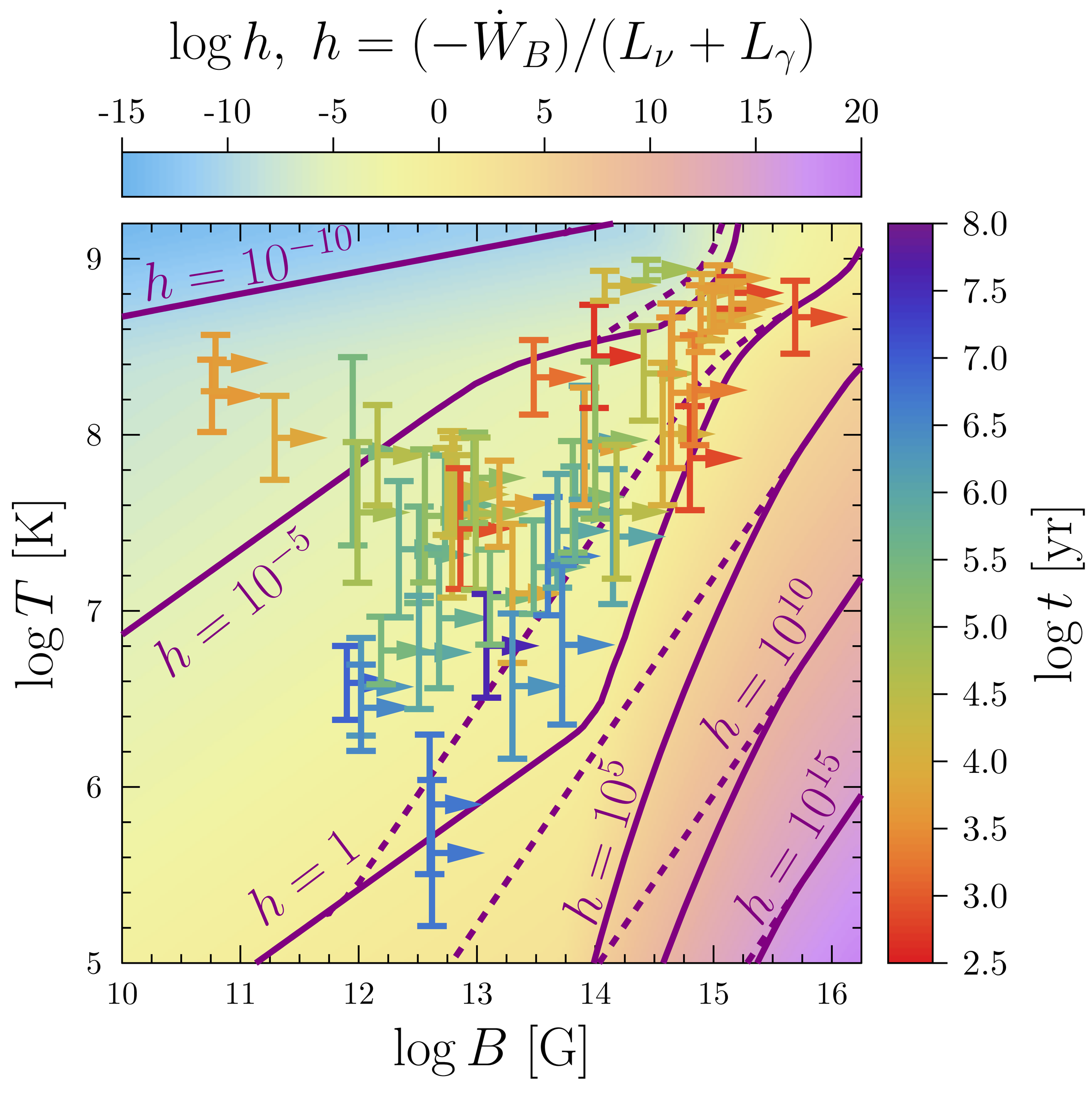


Figure 4: The ratio $h$ of the heating rate and total cooling luminosity as a function of averaged internal field $B$ and internal temperature $T$ (pastel color map). Purple lines indicate the constant $h$ values [solid for eq. (13b), dashed for eq. (13a)]. The rainbow-colored bars are the observed NSs with $T$ estimated from the photon luminosity and $B$ estimated from above as the field at the pole $B_p$. See text for details.

To compare our results to observations, we take observational data mainly from [1] and [11] (with slight updates; the stars with two-sided estimates for $L_\gamma$, field at the pole $B_p$ and age $t$ are considered only). For each NS, we estimate an internal field as $B > B_p$, and derive a range of internal $T$ from observed $L_\gamma$ using eight models of the heat blanketing envelope (differ by composition, surface temperature distribution model, and accounting for Coulomb plasma nonideality). Resulting positions of the observed NSs in the $B - T$ plot are shown in fig. 4 by rainbow-colored error bars (color for age $t$). Fig 4 suggests that (almost) all observed NSs are located within the area $h < 1$. For a given $T$, the most magnetized NSs lie close to the line $h = 1$, i.e. $\dot{W}_B + L_\nu + L_\gamma = 0$. It is especially pronounced for some of magnetars with $t \lesssim 10^4$ yr and old pulsars with $t \sim 10^6 - 10^7$ yr.

## 8. Conclusion

We derive the magnetic field dissipation rate $\dot{W}_B$ in the $npe\mu$ NS core. Instead of precise but cumbersome calculations, we give simple order-of-magnitude estimates for $\dot{W}_B$ in three cases: normal core (eqs. 5 for practical usage), strongly paired $p$, and strongly paired both $n$ and $p$ (eq. 12b). We show that presence/absence of $n$ pairing does not affect $\dot{W}_B$ significantly. Comparison to observations (fig. 4) shows that, for a given internal temperature, the most magnetized NSs should have $\dot{W}_B$ almost balanced by their total cooling luminosity. This indicates importance of core heating for NS cooling studies. Of course, this work has a lot of crude simplifications. Some of them: no account for crustal magnetic heating; expressions used for coefficients $D_\ell$ in eq. (12b) are valid at $B \ll H_{c2}$ only; estimate for the coefficients $D_n$ and $J_{e\mu}$ in the superfluid case are very crude. These lacks make quantitative estimates (5) and (12) accurate only up to a factor $\sim 10$. Refining is left for future work.

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