Refinement of the pulsars classical magnetic field estimator

Anton Biryukov[1,2], Artyom Astashenok[3] and Gregory Beskin[4,2]

[1]SAI MSU, [2]Kazan Fereral U., [3]Kant Baltic Feredal U., [4]SAO RAS

Any feedback is welcome to ant.biryukov@gmail.com

Abstract

A new, robust timing-based formula for the magnetic field estimation B of isolated radiopulsars is suggested. In contrast to the standard "magneto-dipolar" approach, we assume a weak dependence of pulsars spin-down luminosity on their magnetic obliquities $L(\alpha) \propto B^2(1+\sin^2\alpha)$ as it has been derived from numerical simulations of realistic non-vacuum NS magnetospheres. The state-of-the-art constraints on the isolated NS masses and obliquity distributions are taken into account. The representative subset of more than 20 realistic equations of state is also considered. We show that the surface magnetic field of an individual pulsar can be, in principle, constrained with the relative uncertainty $\lesssim 30\%$ at 1σ confidence.

Introduction

Magnetic fields of the *isolated radiopulsars* can be routinely inferred from their timing within a given spin-down law. The classical equation for the latter (written for a spherical neutron star with moment of inertia I and radius R) is

$$P\cdot\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{4\pi^2R^6}{Ic^3}\cdot B^2\cdot f(\alpha) \qquad (1)$$

Here B is the field strength at the *magnetic equator*, α is the angle between the pulsar magnetic and spin axes, while c is the speed of light. This law explicitly defines B as a function of period P, its derivative $\dot{P}$ and the dimensionless function $f(\alpha)$. Many propositions about $f(\alpha)$ structure have been made so far. But the most frequently used one – so-called the "magneto-dipolar" model – presumes $f(\alpha) = 2\sin^2\alpha/3$. Adopting further $\alpha = 90^\circ$, $I = I_0 = 10^{45}$ g cm^2 and $R = R_0 = 10$ km, it provides a classical formula for the estimation of pulsars magnetic fields:

$$B_{\rm md}(P,\dot{P}) = \sqrt{\frac{3I_0c^3}{8\pi^2R_0^6}\cdot P\dot{P}} = 3.2\times10^{19}\sqrt{P\dot{P}}\ \mathrm{Gs} \qquad (2)$$

Even if one disregards the fact that equation (2) is based on the unrealistic assumptions (namely, the vacuum NS magnetosphere and common values of I and R for all the pulsars), the obliquity $\alpha = 90^\circ$ assumed in $B_{\rm md}$ formally makes this estimation only a lower limit of the NS surface field strength.

On the other hand, the direct three-dimensional MHD and particle-in-cell numerical simulations of the oblique pulsars magnetospheres undertaken in recent years [4, 2] have shown that realistic $f(\alpha)$ can be approximated by a simple analytic formula

$$f(\alpha) = k_0 + k_1\sin^2\alpha, \qquad (3)$$

where $k_0 \approx 1$, $k_1 \approx 1.4$ and both of them are constant within the terms proportional to $(R/Pc)^2$. In contrast to the "magneto-dipolar" model, this improved solution for $f(\alpha)$ provides a way to measure the surface magnetic field of observed isolated pulsars with relatively high precision even when α remains unknown at all.

Objectives

- Derive the correction to the classical estimator $B_{\rm md}(P,\dot{P})$ on the basis of the state-of-the-art understanding of the spin-down physics of isolated neutron stars.
- Take into account the existing observational constraints on the isolated neutron star masses M and obliquity α distributions.
- Consider a representative and large enough subset of realistic equations of state (EOS) that do not contradict to the observations in hand.
- Estimate the realistic uncertainties that can be achieved in the timing-based measurements of individual pulsars magnetic fields.

Magnetic field calculus

Extracting B from (1) and adopting (3) with $k_1 = 1$ and $k_2 = 1.4$ for $f(\alpha)$ one gets the value

$$B(M,\alpha,P,\dot{P}) = \sqrt{\frac{c^3}{4\pi^2}}\times\frac{\sqrt{I(M)}}{R^3(M)}\times\sqrt{\frac{P\dot{P}}{1+1.4\sin^2\alpha}} \qquad (4)$$

dependent on the instantaneous α, EOS (i.e. the inertia and the size of the star) and full NS gravitational mass M. Hereafter we focus on the logarithmic correction to the classical formula (2)

$$\Delta_{\rm B} = \Delta_{\rm B}^{\rm (eos)}(M,\alpha) \equiv \log B(M,\alpha,P,\dot{P}) - \log B_{\rm md}(P,\dot{P}), \qquad (5)$$

which describes: (i) the deviation of the actual values of I and R from I_0 and R_0 respectively; (ii) the effect of $\alpha \neq 90^\circ$. Moreover, $\Delta_{\rm B}$ does not depend neither on P nor on $\dot{P}$. But, it is remarkable that while α runs the full interval from 0 to 90 degrees, the correction $\Delta_{\rm B}$ changes only within ±0.1 dex.

Adopting the known distributions of isolated pulsars obliquities, masses (which can be constrained from observations of non-recycled pulsars in binary systems) and one of the realistic EOSes, we are able to calculate the *distribution* of the correction Δ_B. In other words, the latter can be used as a *random* variable

$$\Delta_{\rm B}^{\rm (eos)} \sim p(\Delta_{\rm B}|{\rm eos}) \qquad (6)$$

for given probability densities $p(\alpha)$ and $p(M)$. Moreover, we have found within a simple numerical population synthesis that in fact values of Δ_B are weakly correlated to $\log B_{\rm md}(P,\dot{P})$. Therefore the basic moments of $p(\Delta_{\rm B}|{\rm eos})$ – the average $\langle\Delta_{\rm B}^{\rm (eos)}\rangle$ and standard deviation $\sigma\left[\Delta_{\rm B}^{\rm (eos)}\right]$ – have quite clear physical meaning. In particular,

$$\log B^{\rm (eos)}(P,\dot{P}) = \log B_{\rm md}(P,\dot{P}) + \langle\Delta_{\rm B}^{\rm (eos)}\rangle \qquad (7)$$

provides an *unbiased* estimation of the pulsar magnetic field with the typical uncertainty of order $\sigma[\Delta_{\rm B}^{\rm (eos)}]$ for a given EOS. The equation (7) is the basic theoretical result of our research.

However, the equation of state of a neutron star matter remains formally unknown. But a large number of reasonable theoretical propositions about it have been made so far. Adopting a number of them, one may construct a timing-based magnetic field estimator which is even more generalized than (7). Indeed, let $\{{\rm eos}_i\}$ be a large enough subset of N representative EOS that do not contradict the experimental data. Let also w_i be the weight of the ith EOS ($i = 1..N$) in the list estimating its chances to be realized in nature, so that $\sum w_i = 1$. Then a new random quantity $\Delta_{\rm B}^*$ can be introduced through the mixture probability density

$$p(\Delta_{\rm B}^*) = \sum_i w_i\times p(\Delta_{\rm B}|{\rm eos}_i) \qquad (8)$$

Its average $\langle\Delta_{\rm B}^*\rangle$ and standard deviation $\sigma\left[\Delta_{\rm B}^*\right]$ have the same meanings as the moments of $p(\Delta_{\rm B}|{\rm eos}_i)$. Namely, the quantity $\log B^*(P,\dot{P}) = \log B_{\rm md}(P,\dot{P}) + \langle\Delta_{\rm B}^*\rangle$ also provides an unbiased estimation of the surface magnetic field of a radiopulsar with uncertainty of order $\sigma\left[\Delta_{\rm B}^*\right]$ when no EOS can be absolutely preferred from the list of N possibilities.

Results

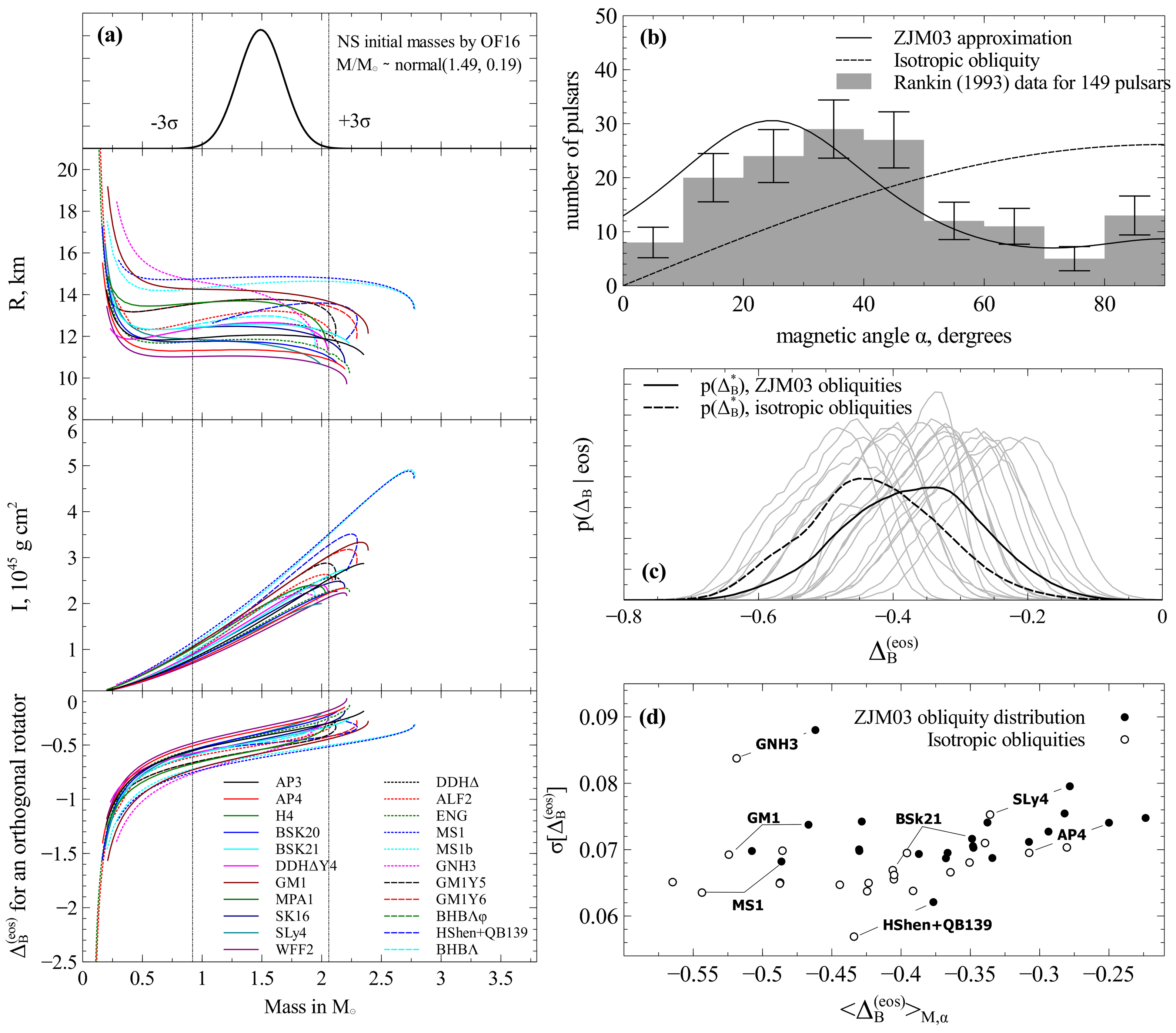

Figure 1: **(a)** Adopted pulsars masses distribution (top panel), R(M) and I(M) relationships for 22 equations of state (middle panels) and logarithmic correction (5) versus pulsar mass assuming $\alpha = 90^\circ$; **(b)** Observed pulsars obliquities distribution based on the data from [3] (grey bars) and its analytical approximation by ZJM03 (solid line) as well as the isotropic obliquities model (dashed line); **(c)** $\Delta_{\rm B}^{\rm (eos)}$ distributions for 22 equations of state (grey lines) within ZJM03 obliquities and generalized $\Delta_{\rm B}^*$ distributions for ZJM03 (solid thick line) and isotropic (dashed thick line) obliquities; **(d)** The moments of $p(\Delta_{\rm B}|{\rm eos})$ distributions for 22 equations of state assuming ZJM03 (filled circles) or isotropic (open circles) obliquities.

We have calculated numerically the probability distributions of $\Delta_{\rm B}^{\rm (eos)}$ and their basic moments – the average and the standard deviation – for the representative list of 22 equations of state which parameters are illustrated in the Figure 1a. The distributions of isolated pulsars masses M and obliquities α have been taken from the models derived in [1] (OF16 hereafter, see the top panel of Fig. 1a) and [5] (ZJM03 hereafter, Fig. 1b) respectively. The results of the calculations are shown in the plots 1(c,d). The shapes of $p(\Delta_{\rm B}|{\rm eos})$ were found to be close to the Gaussian independently on EOS adopted with averages from ≈ -0.51 (for EOS MS1) to ≈ -0.23 (for EOS WFF2). At the same time, their widths appear to be nearly the same for all EOSes

$$\sigma\left[\Delta_{\rm B}^{\rm (eos)}\right] \approx 0.07\pm0.01. \qquad (9)$$

It also has been found that in the case of the isotropic obliquities, $p(\Delta_{\rm B}|{\rm eos})$ generally keep their shapes and widths.

Finally, the distributions of the generalized correction (8), assuming equal weights ($w_i = 1/22$) for all EOS from our list, were also calculated. Their parameters are as follows:

$$\log B^* - \log B_{\rm md} \approx -0.37\pm0.10 \text{ and } \log B^* - \log B_{\rm md} \approx -0.43\pm0.10 \qquad (10)$$

for the ZJM03 and isotropic $p(\alpha)$ respectively. This result means that the timing-based estimation of a pulsar magnetic field can be as precise as ~ 0.1 dex even if neither the equation of state nor the mass nor the obliquity is known. It is an intrinsic property of the adopted pulsars spin-down luminosity model. Being rewritten in a linear form, the quantity

$$B^* \approx \frac{3}{7}B_{\rm md} \qquad (11)$$

provides an unbiased estimation of the actual surface magnetic field strength of an isolated radiopulsar with only ~20-25% uncertainty at 68% confidence level.

Conclusions

- The refined version of the canonical timing-based estimator of the surface magnetic field of normal radiopulsars $B_{\rm md}(P,\dot{P})$ was introduced in a form $\log B = \log B_{\rm md} + \Delta_{\rm B}^{\rm (eos)}(M,\alpha)$. Where M is the NS mass, while α is the magnetic obliquity.
- It was found that within existing observational constraints on masses M and obliquities α of isolated radiopulsars, the value of $\Delta_{\rm B}$ is distributed almost normally with the standard deviation as small as $\approx 0.06..0.09$ dex for most of realistic EOSes. The average value of $\Delta_{\rm B}^{\rm (eos)}$ is, however, nonzero and covers the range from ≈ -0.55 to ≈ -0.25 depending on the choice of EOS.
- The generalized timing-based estimator $\log B^* = \log B_{\rm md} - 0.37\pm0.10$ is also introduced under the assumption of equal chances for all 22 considered equations of state to be realized in nature. It indicates that within the realistic spin-down law the magnetic field of an arbitrary radiopulsar can be estimated with $\lesssim 30\%$ relative error using the timing parameters only.

References

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