Construction and verification of analytical approximation of nonequilibrium neutrino distribution function in core-collapse supernova

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Core-collapse supernova (SN)

- SN matter is opaque for neutrinos ⇒ neutrino interaction with SN matter is an important ingredient of core-collapse supernova models
- Description of neutrino propagation in SNs is required a self-consistent solution of hydrodynamic and neutrino transport equations
- Boltzmann equation for non-equilibrium neutrino distribution function in SN matter is solved only numerically ⇒ it makes difficulties for use of results obtained to other problems

Description of neutrino in a SN

- As a rule, the collapse of the pre-supernova core is isotropic. The neutrino propagates almost spherically symmetric.
- In this case the local nonequilibrium neutrino distribution function $f_\nu(r,\theta,\omega)$ can be described by three parameters at each moment of time:
  r is distance from center of proto-neutron star
  θ is angle between neutrino momentum and radial direction of SN
  ω is neutrino energy
- $f_\nu(r,\theta,\omega)$ can be factorized as
$$f_\nu(r,\omega) \approx N(r)\, \Phi(r,\theta)\, F(r,\omega),$$
  $N(r)$ is the normalized coefficient, which is determined from the neutrino number density
  $\Phi(r,\theta)$ is the neutrino angular distribution
  $F_\nu(r,\omega)$ is the neutrino energy distribution

Analytical approximations of f_ν

We construct two variants of neutrino distribution function, based on an angular representation [1] and well-known spectrum approximations:

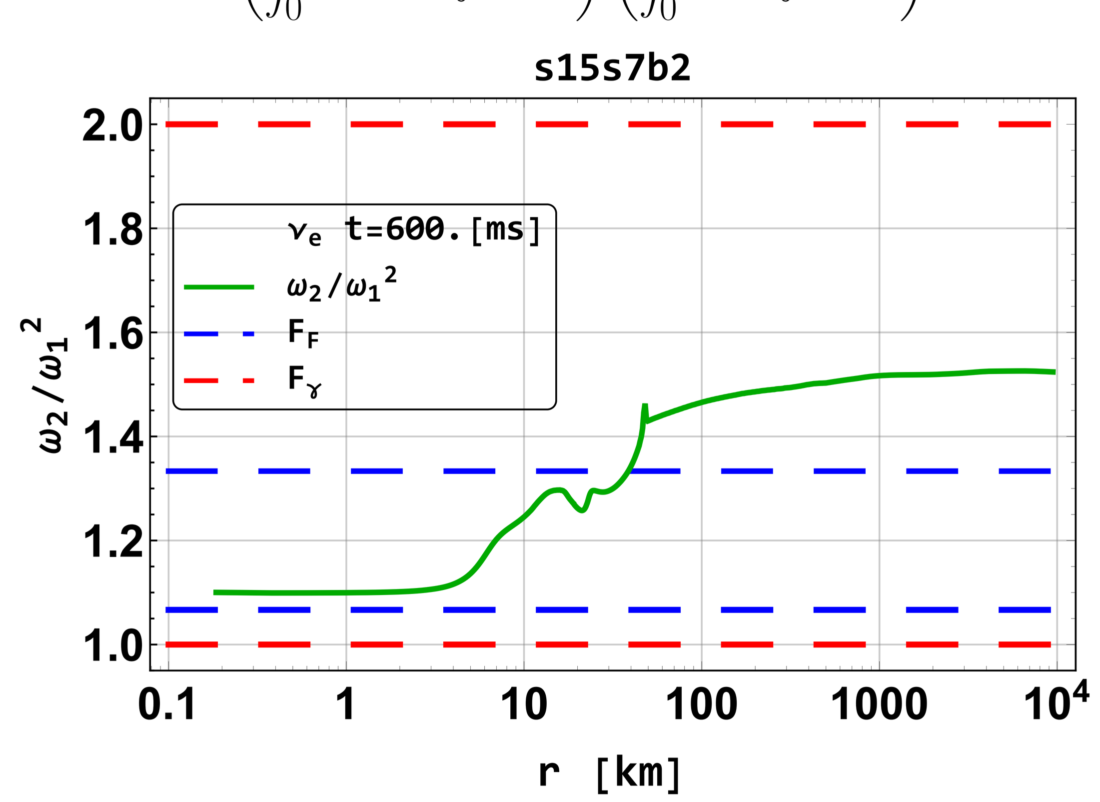
- with nominal Fermi-Dirac approximation [2]:
$$f_\nu^{(F)}(y,\omega) = N_F\, \frac{e^{-(A\,y)^2}}{\exp[\,\omega/T_\nu - a_\nu] + 1},$$
  where $y = 1 - \cos\theta$, A characterizes the angular neutrino distribution, T_ν and a_ν are the nonequilibrium fitting spectral parameters.
- with alpha-fit [3] :
$$f_\nu^{(\gamma)}(y,\omega) = N_\gamma\, e^{-(A\,y)^2} \left(\frac{\omega}{\omega_1}\right)^{\gamma-3} e^{-\gamma\, \omega/\omega_1}.$$
  where ω_1 is the neutrino mean energy and γ represents the amount of spectral pinching.

Both variants of complete neutrino distribution function are 4-parametric. With results of simulations for different types of neutrino, these parameters were found as functions of the distance from SN center and time after a bounce.

Restriction on use Fermi-Dirac approximation

In our analysis we use the results of neutrino distributions from simulations of SN explosion. The simulations were performed with the 1D version of the PROMETHEUS-VERTEX code [4].

We use first two energy moments ω_1 and ω_2 to define the parameters of the used spectral approximations Fermi-Dirac fit and alpha-fits:
$$\omega_n = \left(\int_0^\infty \omega^{n+2} f_\nu\, d\omega\right)\left(\int_0^\infty \omega^2 f_\nu\, d\omega\right)^{-1}.$$

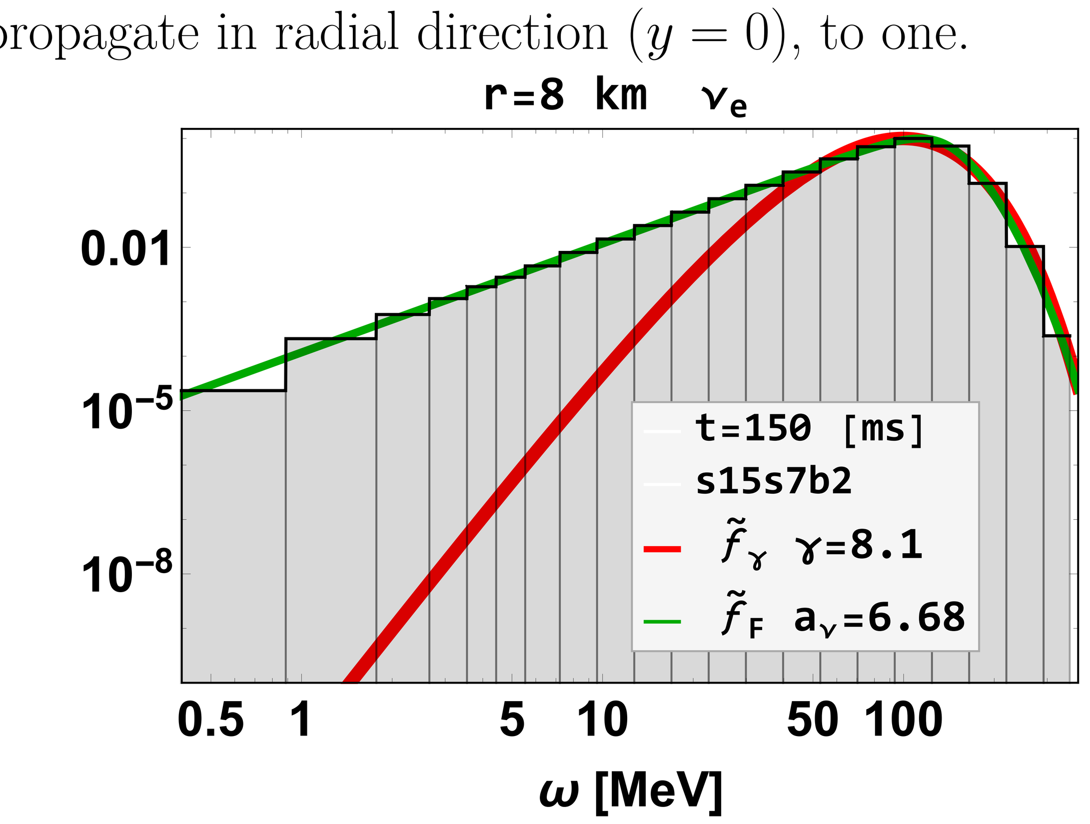


As analyze show, the ratio ω_2/ω_1^2 cannot be arbitrary. We obtain the following ranges for Fermi-Dirac fit $\omega_2/\omega_1^2 \in [16/15, 4/3]$ and for alpha-fit $\omega_2/\omega_1^2 \in [1,2]$, correspondingly. The boundaries of these ranges are shown on the plots: blue lines for Fermi-Dirac fit, red lines for alpha-fit. Green line corresponds to the relation of moments from the simulation data. The relation of moments in the outer region of the supernova exceeds the upper limit of the range for Fermi-Dirac.

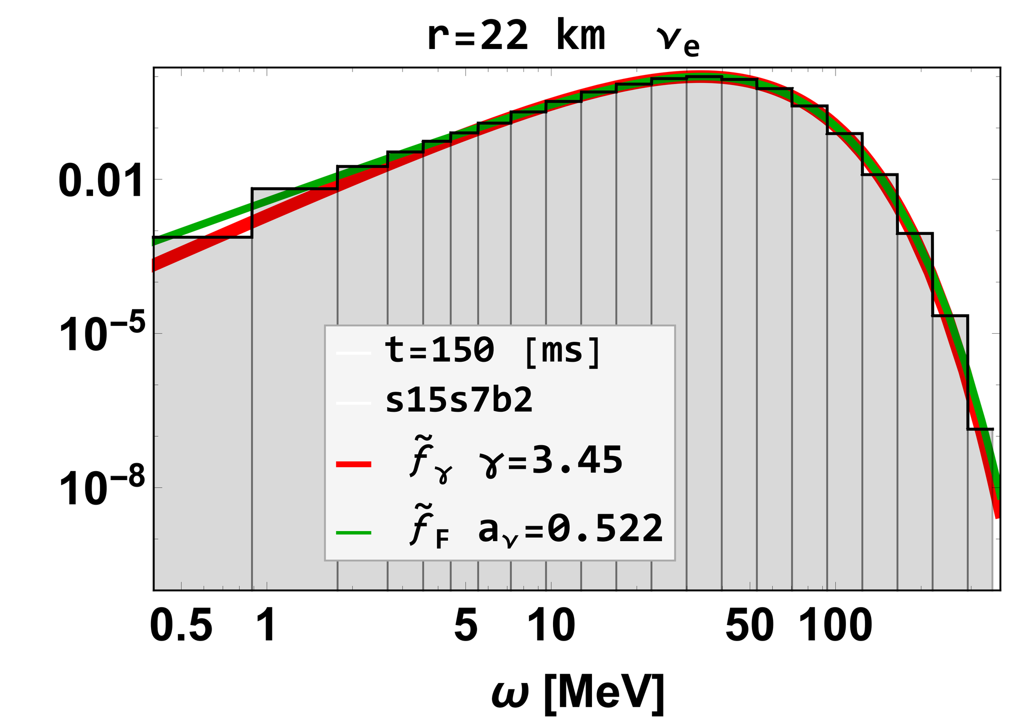
Thus, the parameters of nominal Fermi-Dirac approximation can't be obtained in most of supernova from two first energy moments. However, the other ways could be used for parameters finding.

Comparison of numerical data with spectral approximations

We normalize the spectrum $\tilde{f}$ of neutrino, which propagate in radial direction ($y = 0$), to one.



At a distance of 8 km, where the neutrino radiation is in local equilibrium with the supernova matter, the Fermi-Dirac fit approximates the neutrino spectrum better than other one. However, the alpha-fit differs from data only in low-energy part of spectra, which contain small fraction of all neutrino. At a distance of 22 km, where the neutrino gas is less degenerate,



the alpha-fit describes the neutrino spectrum more better.

Conclusions

The analytical approximations of neutrino distribution function are verified using the results of a 1D simulation of neutrino propagation. It was shown that the approximation, based on a nominal Fermi-Dirac distribution of neutrino spectrum, agrees with results of simulation only in the inner parts of the supernova. Whereas the approximation based on alpha-fit of neutrino energy distribution is more general and has no restrictions for application in any part of the supernova.

References

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